

EEE5109 Digital Filters

Lecture 1

1 Summary

This lecture will focus on:

1. Introduction to discrete time signals
2. Basic discrete time signals
3. Introduction to discrete time systems
4. System properties

2 Introduction to discrete time signals

Digital signal processing techniques play an important role in most modern electronic systems. For example sophisticated signal processing algorithms allow us to hear a caller on the other end of a telephone line even in noisy environments and are used in speech and speaker recognition systems. Signal processing algorithms are also employed in modern wireless communication systems.

Signal processing involves the manipulation of digital signals to achieve desired outcomes. For example we may wish to reduce the amount of audible noise in a digital recording. Signal processing techniques are also useful in extracting information from signals. For example speech signals contain information about both what the speaker is saying and the identity of the speaker. Using an appropriate transform of the speech signal, we can determine what the speaker is saying (speech recognition) and who the speaker is (speaker recognition).

2.1 Signals

Signals are the information bearing quantities that enable the transfer of information. Mathematically we define a signal as an information bearing function of one or more variables. For example a speech signal is a one dimensional function of time that conveys information. An image is a two dimensional function of position. The pixel value at a given position represented by its x and y coordinates carries the information. Figure 1 shows a speech signal viewed in both time (bottom panel) and frequency domain. We will learn more about frequency domain representations in this course.

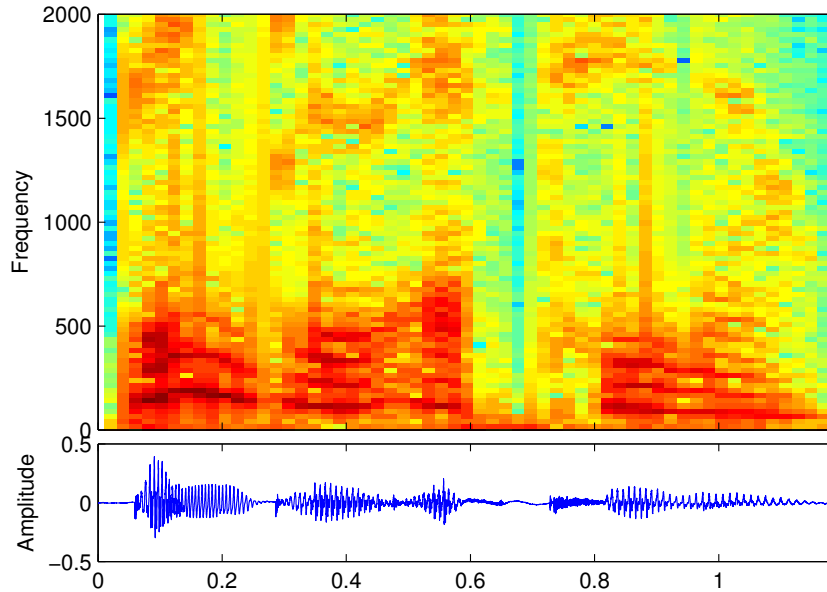


Figure 1: A speech signal viewed in both time (bottom panel) and frequency domain.

2.2 Signal classification

Signals can be classified based on different criteria. For our purposes, the most important classifications are

1. Continuous time and discrete time signals: A Continuous time signal $x(t)$ is defined for all time t . For example body and ambient temperature are continuous time signals. Discrete time signals are defined at discrete times only indexed by the intergers. Thus a discrete time signal is a sequence of numbers where the n th number is denoted by $x[n]$. These signals often arise from the sampling of a continuous time signal $x(t)$ at regular intervals. Therefore

$$x[n] = x(nT_s)$$

T_s is known as the sampling period. The sampling frequency is $\frac{1}{T_s}$. Figure 2 shows a continuous time signal (a) and the discrete time signal obtained from sampling the continuous signal at regular intervals. Two different time domain signal can result in the same discrete time signal after sampling. This means that an appropriate sampling rate must be chosen to allow reconstruction of signals. We will explore this further in the course.

2. Deterministic and random signals: A deterministic signal is completely specified as a function of time. Random signals can be viewed as being drawn from a group of signals. The particular signal drawn is subject to chance. Many real world signals have an element of uncertainty and are best modeled as random. Figure 3 illustrates the ideas of drawing a random signal from a group.
3. Periodic and non-periodic signals: A signal $x(t)$ is said to be periodic if there exists a number T such that

$$x(t) = x(t + T).$$

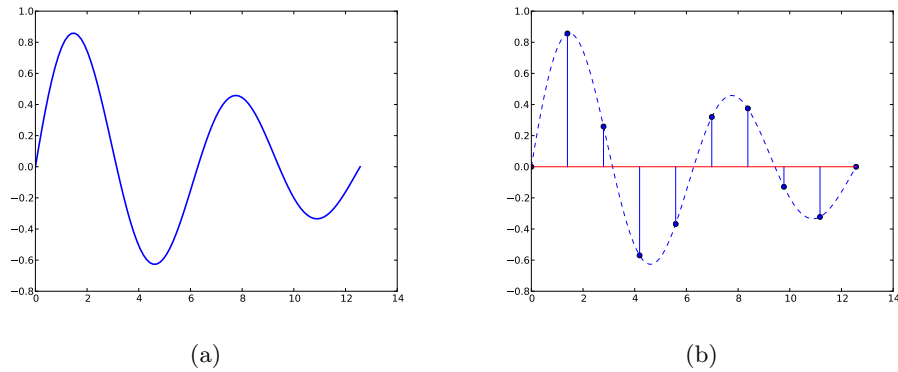


Figure 2: Continuous time signal (a) and the discrete time signal obtained from sampling the continuous signal at regular intervals.

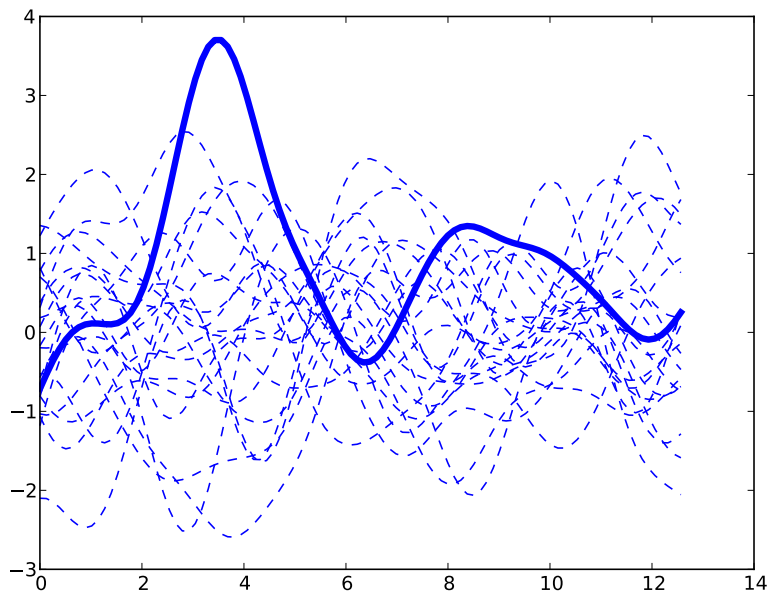


Figure 3: A group of signals (dashed lines) from which a random signal (solid line) is drawn .

The smallest value T for which this relation holds is known as the *fundamental period*. A discrete time signal $x[n]$ is periodic if there is a positive integer N such that

$$x[n] = x[n + N]$$

Example $x[n] = \cos(\pi n/4)$ has a period of $N = 8$. Non-periodic or aperiodic signals are not periodic.

2.3 Basic Sequences

The following basic discrete time signals are important in the study of discrete time signals and systems.

1. The unit sample sequence is defined as

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

This signal plays a very important role in signal processing. Any discrete signal can be expressed in terms of the unit sample. We have

$$x[n] = \sum_{k=-\infty}^{k=\infty} x[k]\delta[n-k]$$

2. The unit step is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

3. The unit ramp is defined as

$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

4. Exponential sequences have the general form

$$x[n] = A\alpha^n$$

where A and α are complex in general.

5. Sinusoidal sequences take the form

$$x[n] = A \cos(\omega_0 n + \phi) \quad \forall n$$

where A and ϕ are real constants.

3 Discrete-time systems

A discrete time system transforms an input sequence $x[n]$ into an output sequence $y[n]$. Mathematically the system is represented by a formula for computing the output sequence from the input sequence. Some examples include

- The delay system $y[n] = x[n - n_d]$ where n_d is a positive integer
- The moving average system defined by the following equation

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$$

The n th output sample is the average of the $M_1 + M_2 + 1$ samples around it. This serves to smooth out abrupt changes.

3.1 Linear systems

Linear systems satisfy two properties namely

1. **Superposition:** if the input sequence $x_1[n]$ produces the output sequence $y_1[n]$ and input $x_2[n]$ produces output $y_2[n]$. Then the output of the system in response to input $x_1[n] + x_2[n]$ is $y_1[n] + y_2[n]$.
2. **Homogeneity:** If input $x[n]$ produces output $y[n]$, then input $ax[n]$ where $a \in \mathbf{C}$ produces output $ay[n]$.

We can say that the input $a_1x_1[n] + a_2x_2[n]$ produces the output $a_1y_1[n] + a_2y_2[n]$. In general we can write that the input

$$x[n] = \sum_k a_k x_k[n]$$

produces the output

$$y[n] = \sum_k a_k y_k[n]$$

where $y_k[n]$ is the response to $x_k[n]$.

3.2 Time invariant systems

A system is said to be time invariant if a delay in the input sequence produces the same delay in the output sequence. Also, the properties of the system do not change with time and therefore the response does not depend on when the input is applied. Formally, if the response to $x[n]$ is $y[n]$, then the response to $x[n - n_0]$ is $y[n - n_0]$.

3.3 Other system properties

1. **Stability:** A system is said to be *bounded input bounded output* (BIBO) stable if every bounded input produces a bounded output. For a system to be of engineering use it must be stable. Stability is explored in detail in most control systems courses.
2. **Causality:** A system is causal if the present output depends only on present and past inputs. Non-causal systems can be implemented in software for example some filters require future values to obtain estimates of present values. These systems do not operate in real time.

Homework 1

1. Read the following sections of Chapter 1 of the course text, Simon Haykin and Barry Van Veen, *Signals and Systems*, 2nd edition, John Wiley and Sons: 1.1, 1.2, 1.3, 1.4, 1.5 and 1.6 paying special attention to the sections on discrete time signals.
2. Using Matlab or Python, produce a stem plot the function

$$x[n] = \cos\left(\frac{\pi n}{4}\right).$$

and verify that $x[n]$ is periodic with period $N = 8$.

3. A discrete time signal $x[n]$ is obtained by sampling the signal $x(t) = \sin(2\pi t)$ with a sampling period $T_s = \frac{1}{4}$. What are the values of $x[n]$ for $-3 \leq n \leq 3$. Can you come up with a general formula for $x[n]$? Using Matlab or Python plot $x(t)$ and superimpose a stem plot of $x[n]$ on the same figure.
4. For the delay system $y[n] = x[n - n_d]$ where n_d is a positive integer. Sketch the output of the system when $n_d = 5$ and $x[n] = u[n]$, the unit step.