EEE 5109 Digital Signal Processing.

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Today's Lecture

1. Linear systems

General expression for discrete signals

- Consider the signal

$$x[n] = \begin{cases} 2 & n = -1 \\ -3 & n = 0 \\ 1 & n = 3 \\ 0 & \text{otherwise} \end{cases}$$

- $x[n] = 2\delta[n+1] - 3\delta[n] + \delta[n-3]$
- In general

$$x[n] = \sum_{k=-\infty}^{k=\infty} x[k]\delta[n-k]$$

Discrete-time systems

- The system is described as a transformation that maps an input sequence into an output sequence

$$y[n] = T\{x[n]\}$$

Linear systems

- Linear systems satisfy two properties namely
    1. Superposition: if the input sequence $x_1[n]$ produces the output sequence $y_1[n]$ and input $x_2[n]$ produces output $y_2[n]$. Then the output of the system in response to input $x_1[n] + x_2[n]$ is $y_1[n] + y_2[n]$.
    2. Homogeneity: If input $x[n]$ produces output $y[n]$, then input $ax[n]$ where $a \in \mathbf{C}$ produces output $ay[n]$.

Examples

- Consider an accumulator system whose response $y[n]$ to an input $x[n]$ is given by

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Derive and sketch the output of the accumulator to

1. $u[n]$, the unit step
2. $r[n]$, the unit ramp

Time invariant systems

- A system is said to be time invariant if a delay in the input sequence produces the same delay in the output sequence. Also, the properties of the system do not change with time and therefore the response does not depend on when the input is applied. Formally, if the response to $x[n]$ is $y[n]$, then the response to $x[n-n_0]$ is $y[n-n_0]$.

Other system properties

- *Stability*: A system is said to be *bounded input bounded output* (BIBO) stable if every bounded input produces a bounded output. For a system to be of engineering use it must be stable. Stability is explored in detail in most control systems courses.
- *Causality*: A system is causal if the present output depends only on present and past inputs. Non-causal systems can be implemented in software for example some filters require future values to obtain estimates of present values. These systems do not operate in real time.