EEE5108/ETI5103 Digital Signal Processing.

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Today's Lecture

DTFT Properties

- Linearity: If $x_1[n] \leftrightarrow X_1(\omega)$ and $x_2[n] \leftrightarrow X_2(\omega)$, then $a_1 x_1[n] + a_2 x_2[n] \leftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)$
- Time shifting: If $x[n] \leftrightarrow X(\omega)$ then $x[n - n_0] \leftrightarrow X(\omega) e^{-j\omega n_0}$.
- Modulation: If $x[n] \leftrightarrow X(\omega)$ then $x[n] e^{j\omega_0 n} \leftrightarrow X(\omega - \omega_0)$.
- Convolution: If $x[n] \leftrightarrow X(\omega)$ and $h[n] \leftrightarrow H(\omega)$ then $x[n] * h[n] \leftrightarrow X(\omega) H(\omega)$
- Windowing: If $x[n] \leftrightarrow X(\omega)$ and $w[n] \leftrightarrow W(\omega)$ then $x[n] w[n] \leftrightarrow \frac{1}{2\pi} X(\omega) * W(\omega)$

Parseval's theorem

- The energy of a signal is defined as

$$E_x = \sum_{n=-\infty}^{\infty} x[n]x^*[n] = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- Parseval's theorem states that

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

Inverse DTFT

- We have

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

where

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Example

- The ideal low pass filter has the frequency response

$$H_{lp}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases}$$

- We can derive $h_{lp}[n]$ via the inverse DTFT

Example

- The ideal high pass filter has the frequency response

$$H_{hp}(\omega) = \begin{cases} 0 & |\omega| < \omega_c \\ 1 & \omega_c < |\omega| \leq \pi \end{cases}$$

- We have $H_{hp}(\omega) = 1 - H_{lp}(\omega)$
- We can then derive $h_{hp}[n]$