EEE5108/ETI5103 Digital Signal Processing.

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Today's Lecture

1. z-Transform

z-Transform

- Recall that for a discrete time sequence $x[n]$, its Fourier transform $X(e^{j\omega})$ is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{1}$$

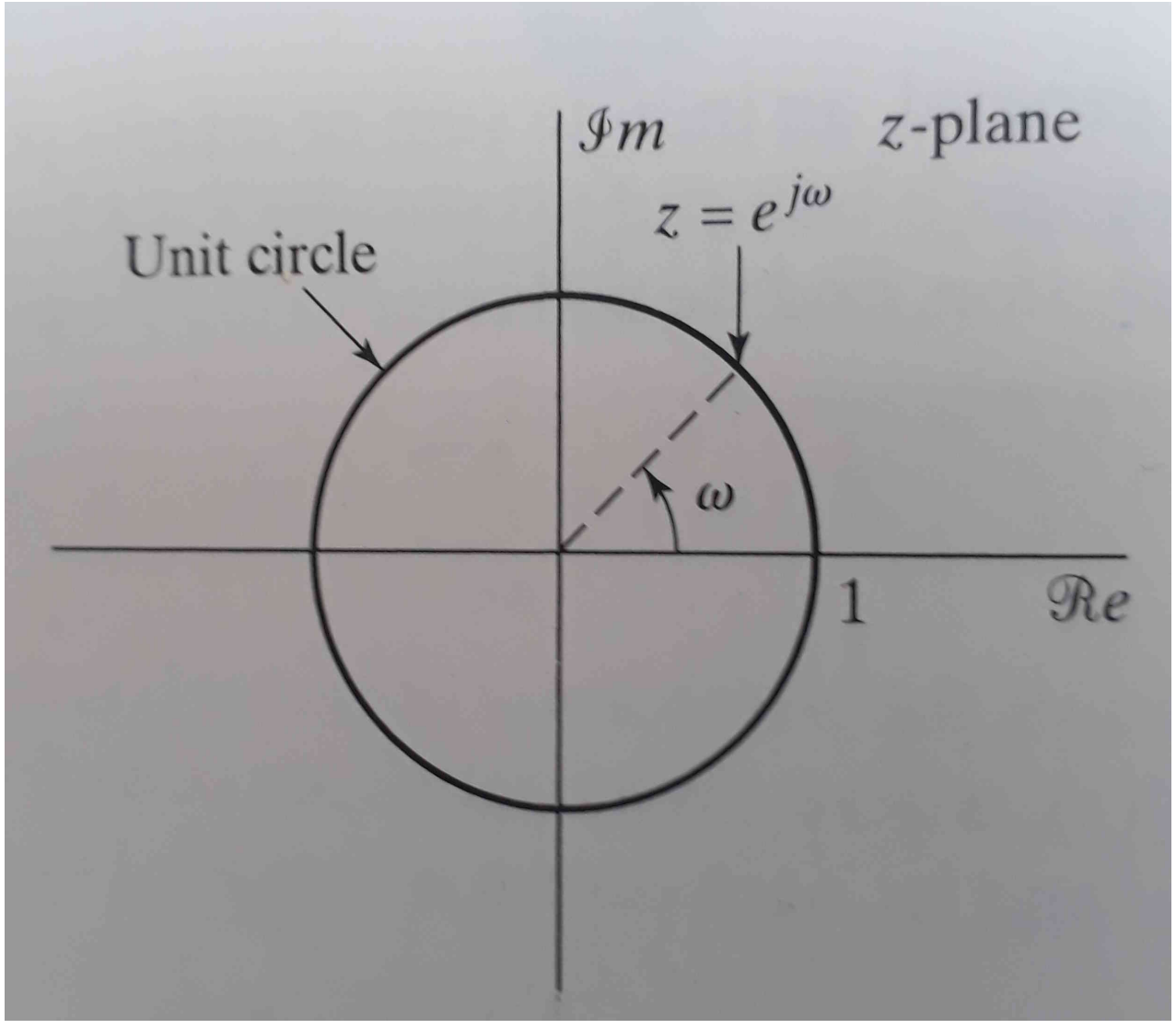
- The z-transform is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{2}$$

- When the Fourier transform exists, it is $X(z)$ with $z = e^{j\omega}$

z-Transform

- z is a complex variable
- We can express z in polar form as $z = re^{j\omega}$
- When we evaluate the z-transform along the unit circle $r = 1$, we get the Fourier tranform


$\mathcal{I}m$
z-plane
$z = e^{j\omega}$
Unit circle
ω
1
$\mathcal{R}e$

Example

- Consider the sequence $x[n] = a^n u[n]$. Determine its z-transform

Solution

-

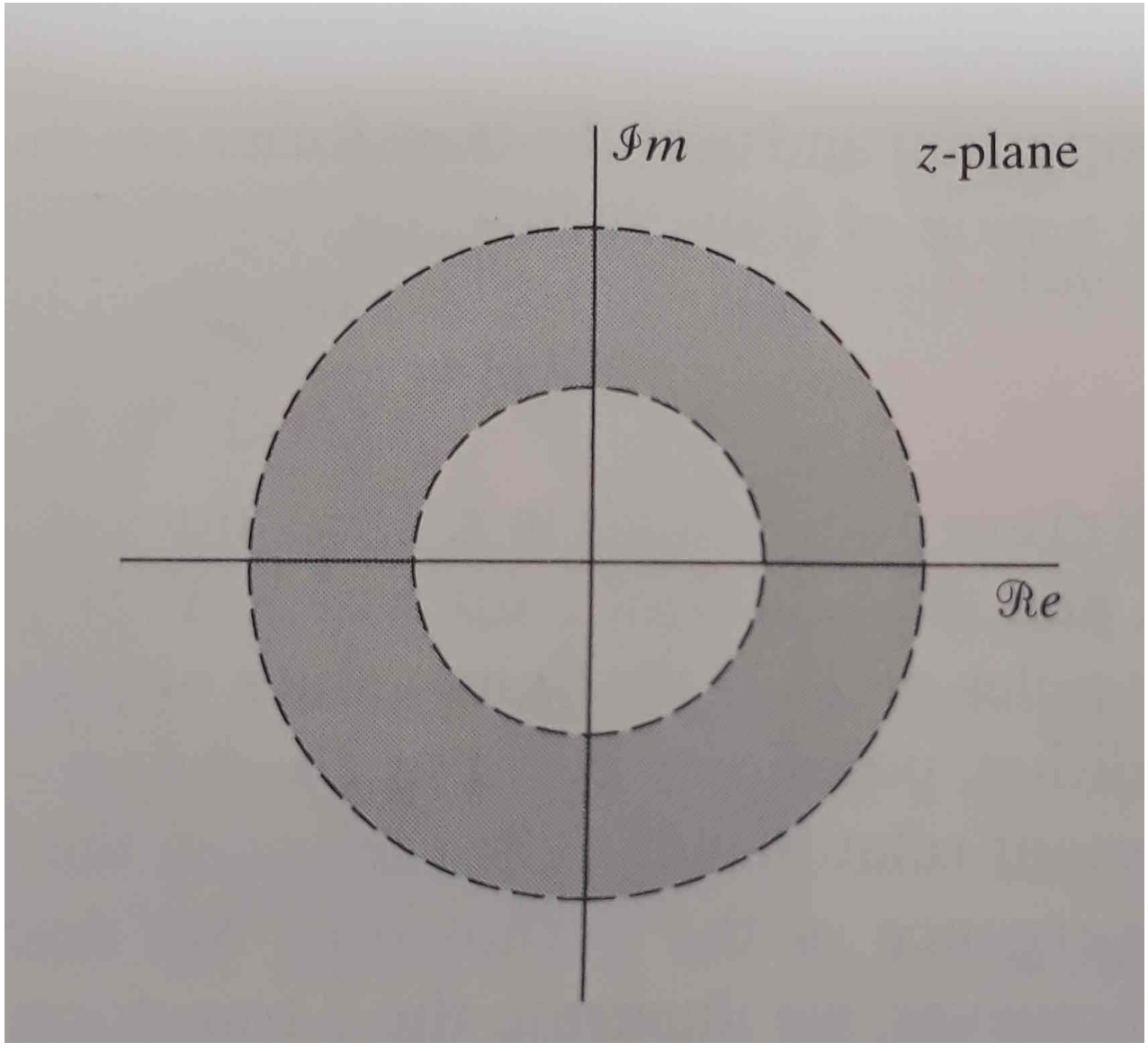
$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} \\ &= \sum_{n=0}^{\infty} a^n z^{-n} \\ &= \sum_{n=0}^{\infty} (az^{-1})^n \\ &= \frac{1}{1 - az^{-1}} \end{aligned}$$

- This converges for $|az^{-1}| < 1$ or $|z| > |a|$

Region of Convergence

- $X(z)$ converges when $|X(z)| < \infty$
- We have

$$\begin{aligned} |X(z)| &= \left| \sum_{n=-\infty}^{\infty} x[n] z^{-n} \right| \\ &\leq \sum_{n=-\infty}^{\infty} |x[n]||(re^{j\omega})^{-n}| \\ &\leq \sum_{n=-\infty}^{\infty} |x[n] r^{-n}| < \infty \end{aligned}$$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