Signals and Communication Lecture 1

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1 Summary

This lecture will focus on:

1. Introduction to signals and communication systems
2. Elementary signals
3. Operations on signals

2 Introduction to signals and communication systems

Communication is the transfer of information from one point to another. Signals are the information bearing quantities that enable this transfer of information. Mathematically we define a signal as an information bearing function of one or more variables. For example a speech signal is a one dimensional function of time that conveys information. An image is a two dimensional function of position. The pixel value at a given position represented by its x and y coordinates carries the information. Figure 1 shows a speech signal viewed in both time (bottom panel) and frequency domain. We will learn more about frequency domain representations in this course.

2.1 Signal classification

Signals can be classified based on different criteria. For our purposes, the most important classifications are

1. Continuous time and discrete time signals: A Continuous time signal $x(t)$ is defined for all time t. For example body and ambient temperature are continuous time signals. Discrete time signals are defined at discrete times only indexed by the integers. Thus a discrete time signal is a sequence of numbers where the nth number is denoted by $x[n]$. These signals often arise from the sampling of a continuous time signal $x(t)$ at regular intervals. Therefore

$$x[n] = x(nT_s)$$

T_s is known as the sampling period. The sampling frequency is $\frac{1}{T_s}$. Figure 2 shows a continuous time signal (a) and the discrete time signal obtained from sampling the continuous signal at regular intervals. Two different time domain signal can result in the same discrete time signal after sampling. This means that an appropriate sampling rate must be chosen to allow reconstruction of signals. We will explore this further in the course.

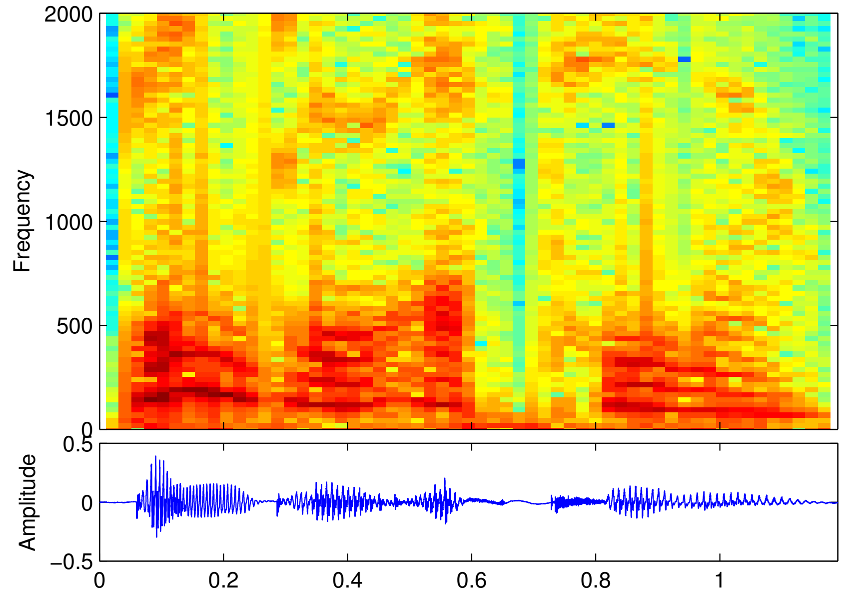


Figure 1: A speech signal viewed in both time (bottom panel) and frequency domain.

2. Even and odd signals: For an even signal we have

$$x(-t) = x(t) \quad \forall t$$

For an odd signal we have

$$x(-t) = -x(t) \quad \forall t$$

Exercise: Give examples of odd and even signals.

All signals can be decompose into a sum of an odd and even component. That is

$$x(t) = x_e(t) + x_o(t)$$

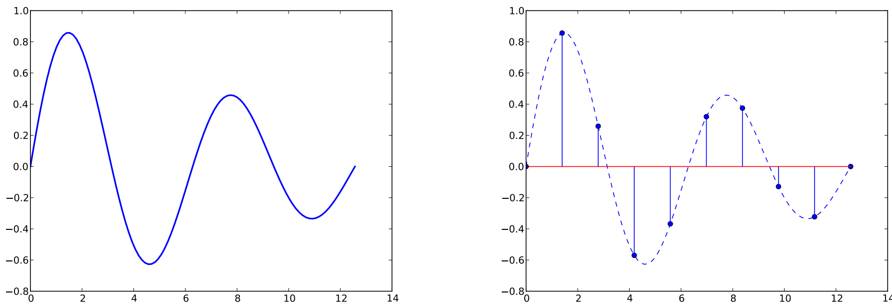
where $x_e(t)$ is the even component of $x(t)$ and $x_o(t)$ is the odd component. We can show that

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

and

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

3. Deterministic and random signals: A deterministic signal is completely specified as a function of time. Random signals can be viewed as being drawn from a group of signas. The particular signal drawn is subject to chance. Many real world signals have an element of uncertainty and are best modeled as random. Figure 3 illustrates the ideas of drawing a random signal from a group.



(a) (b)

Figure 2: Continuous time signal (a) and the discrete time signal obtained from sampling the continuous signal at regular intervals.

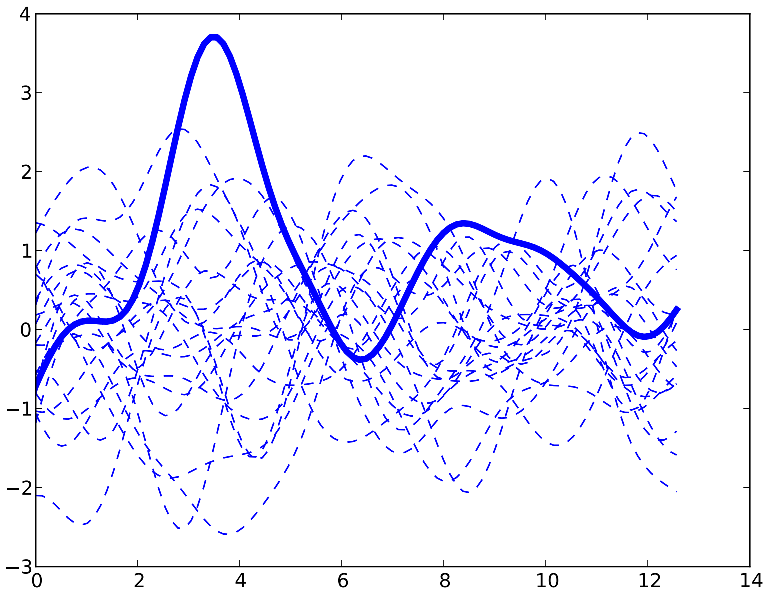


Figure 3: A group of signals (dashed lines) from which a random signal (solid line) is drawn .

4. Periodic and non-periodic signals: A signal $x(t)$ is said to be periodic if there exists a positive constant T such that

$$x(t) = x(t + T).$$

The smallest value T for which this relation holds is known as the *fundamental period*. For each of the following signals, state whether they are periodic and of so determine the period

- $\cos(2\pi t)$
- $\cos^2((2\pi t)$
- $\sin(t)$

A discrete time signal $x[n]$ is periodic is there is a positive integer N such that

$$x[n] = x[n+N]$$

Example $x[n] = \cos(\pi n/4)$ has a period of $N = 8$. Non-periodic or aperiodic signals are not periodic.

2.2 Basic signals

In systems analysis and signal processing, it is sometimes useful to express signals and linear combinations of more basic signals. Then if these signal is input to a linear system the output will be a linear combination of responses of the system to the basic signal.

2.2.1 Basic continuous time signals

1. The Dirac delta pulse is defined as follows

$$\delta(t) = 0, t \neq 0$$

and

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

The sifting property will be useful when we explore sampling.

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$$

The Dirac delta pulse can be seen as the limit of

$$p_\Delta(t) = \begin{cases} \frac{1}{\Delta} & |t| < \frac{\Delta}{2} \\ 0 & \text{Otherwise} \end{cases}$$

as $\Delta \rightarrow 0$. Question: Is the Dirac delta function even or odd?

2. The unit step is defined as

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

3. The unit ramp

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

These basic signals are related...

$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

$$r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

Example: express a rectangular and triangular pulse train as a sum of basic signals.

2.3 Systems

A system operates on input signals to produce desired output signals. In communication systems the input signal is manipulated at the *transmitter* to enable its transmission over a medium (the *channel* for example free space in broadcast TV or optical fiber). The signal must then be recovered at the *receiver* with minimal distortion.

2.4 Linear systems

The analysis of communication systems involves techniques learned in an introductory systems course. Here we briefly review linear time invariant (LTI) systems. Linear systems satisfy two properties namely

1. Superposition: if input $x_1(t)$ produces output $y_1(t)$ and input $x_2(t)$ produces output $y_2(t)$. Then the output of the system in response to input $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$.

2. Homogeneity: If input $x(t)$ produces output $y(t)$, then input $ax(t)$ where $a \in \mathbf{C}$ produces output $ay(t)$.

A system is said to be time invariant if a delay in the input produces the same delay in the output. Also, the properties of the system do not change with time and therefore the response does not depend on when the input is applied. Formally, if the response to $x(t)$ is $y(t)$, then the response to $x(t - \tau)$ is $y(t - \tau)$ where τ is any real number.

Homework 1

1. Read about operations on signals. Time reversal, time scaling and delay
2. Read about the following classifications of signals. Even vs Odd signals, Energy vs Power signals
3. Using Matlab or Python, produce a stem plot of $x[n] = \cos(\pi n/4)$, verify that $N = 8$
4. Compute $\int_a^b x \sin(cx) dx$
5. Sketch the function given by $r(t) - r(t-1) + r(t-5)$ where r(t) is the unit ramp.