Signals and Communication II.

Dr. Ciira Maina ciira.maina@dkut.ac.ke

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Noise in communication systems

- Noise is a broad term referring to unwanted signals that interfere with the transmission of information signals.
- There are several sources of noise such as random motion of electrons and the discrete nature of charge

Due to the random nature of noise, we use statistical techniques to characterise and analyse noise.

Statistical characterisation of noise

Let v_n(t) represent a noise voltage, the average value of this signal is zero and is given by

$$\overline{v_n}(t) = \langle v_n(t) \rangle = \frac{1}{T} \int_T v_n(t) dt = 0$$
 (1)

The noise variance or equivalently the noise power is non zero.

$$\overline{v_n(t)^2} = \frac{1}{T} \int_T v_n^2(t) dt \neq 0$$
⁽²⁾

The root mean square noise voltage is given by

$$v_{n,rms} = \sqrt{\overline{v_n(t)^2}} \tag{3}$$

Power spectrum

- The power spectrum of noise characterises the noise power as a function of frequency.
- The power spectrum is related to the autocorrelation function of the noise signal

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- Autocorrelation is a mathematical way of quantifying how similar nearby samples of a signal are.
- Signals can vary from totally unpredictable to constant

Correlation

- Noise signals are examples of random signals which cannot be predicted exactly at a given time
- ► We characterise them by self-similarity. Similarity between a sample at a time t and a time t + τ

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See Notebook for more details.

Thermal Noise

The noise power generated by a resistor R is represented by a voltage source in series with the resistor. The mean square value of the resistor is

$$\overline{v_n^2} = 4k_B TRB \tag{4}$$

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- ▶ Where k_B is Boltzmann's constant, T is the temperature in Kelvin and B is the bandwidth.
- This noise is characterised by a flat spectrum and is called 'white' noise. The autocorrelation function is a Dirac delta function.

Example

- Let R = 10kΩ, B = 10⁶MHz and T = 20C. Compute the RMS noise voltage.
- If we have two resistors in series, then the mean square noise voltage is

$$\overline{v_n^2} = 4k_B T (R_1 + R_2) B = \overline{v_{n1}^2} + \overline{v_{n2}^2}$$
(5)

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- The noise powers add, NOT the noise voltages
- For general circuits we use the Thevenin's equivalent circuit to determine the noise voltage.

Example

• Determine $V_{T,s}$, R_T and $\overline{v_{Tn}^2}$



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